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Title	Optical wave propagation simulation, Wigner phase-space diagrams, and wave energy confinement
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Publication date	2005-10-16
Publication information	Frontiers in Optics, OSA Technical Digest Series
Conference details	Paper presented at Frontiers in Optics (FiO), Tucson, Arizona, October 16, 2005
Publisher	Optical Society of America
Link to online version	http://www.opticsinfobase.org/abstract.cfm?URI=FiO-2005-FTuM4
Item record/more information	http://hdl.handle.net/10197/3475
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Optical wave propagation simulation, Wigner phase-space diagrams, and wave energy confinement

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Abstract: The number of samples required for efficient numerical simulation of wave propagation can be determined by a combination of Wigner phase-space techniques, wave energy confinement arguments, and a theorem relating energy confinement to accuracy.

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OCIS Codes: (050.1940) Diffraction; (070.2580) Fourier optics

The number N of samples u_n required for numerical simulation of the propagation of a monochromatic optical wave $u(x)$ is often expressed in terms of the space-bandwidth product of the wave [1],

$$N = W_o B_o, \quad (1)$$

where $u(x) \equiv 0$ for $|x| \geq W_o/2$ and $U(v) \equiv 0$ for $|v| \geq B_o/2$, with $U(v) = \int u(x) \exp(-i2\pi vx) dx$, the Fourier transform of $u(x)$ (one-dimensional notation is used, extensions to two dimensions being straight-forward).

There are two difficulties associated with Eq. (1) and its application: (a) the definitions of W_o and B_o generally lack precision, especially since it is mathematically impossible for both $u(x)$ and $U(v)$ to have compact support; and (b) the effect of too few samples, as manifested by aliasing in the reconstruction of $u(x)$ from a set of discrete samples, is difficult to quantify. Propagation of $u(x)$ introduces additional complications since, through diffraction, it leads to a spreading of the wave—and, thus, to an increase in spatial extent $W(z)$ —that is often difficult to quantify. As noted in Ref. [1], the Wigner phase-space diagram can provide insight into the spreading of a wave as it propagates, but it does not provide a clear means for specifying the sample rate and number of samples appropriate for a given wave $u(x)$ and propagation distance z .

This paper has two objectives: (1) to provide means for selecting W_o and B_o in Eq. (1) that relates quantitatively to errors in the reconstruction of $u(x)$ from its sample values, and (2) to present a means for specifying the number of samples $N(z)$ required for numerical propagation of $u(x)$ through distance z , for $z_1 < z < z_2$, in the case where $|u(x, z_1)|$ and $|u(x, z_2)|$ are both known.

The paper proceeds as follows. First, the width W_o and spatial frequency bandwidth B_o of wave $u(x)$ are defined in terms of a fraction-of-signal-energy metric η . Next, it is shown that the fractional mean-square error in the continuous reconstruction $\tilde{u}(x)$ obtained from signal samples u_n is expressible in terms of η . Bounds on $\eta(z)$ are then established for propagation distance z satisfying $z_1 \leq z \leq z_2$, first for a special “light tube” case [2], then for a general case. The application of the energy-confining “light tube” to efficient numerical simulation of wave propagation is then discussed.

[1] A. W. Lohmann, R. G. Dorsch, D. Mendlovic, Z. Zalevsky, and C. Ferreira, “Space-bandwidth product of optical signals and systems,” *J. Opt. Soc. Am. A*, Vol. 13, pp. 470-473 (1996).

[2] William T. Rhodes, “Light Tubes, Wigner Diagrams, and Optical Wave Propagation Simulation,” in *Optical Information Processing: A Tribute to Adolf Lohmann*, H. John Caulfield, Editor (SPIE Press, Bellingham, 2002), Chapter 15 (pp. 343-356).