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Optical wave propagation simulation, Wigner phase-space diagrams, and wave energy confinement

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Abstract: The number of samples required for efficient numerical simulation of wave propagation can be determined by a combination of Wigner phase-space techniques, wave energy confinement arguments, and a theorem relating energy confinement to accuracy.

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The number N of samples u_n required for numerical simulation of the propagation of a monochromatic optical wave $u(x)$ is often expressed in terms of the space-bandwidth product of the wave [1],

$$N = W_o B_o, \tag{1}$$

where $u(x) \cong 0$ for $|x| \geq W_o/2$ and $U(\nu) \cong 0$ for $|\nu| \geq B_o/2$, with $U(\nu) = \int u(x)\exp(-i2\pi\nu x)dx$, the Fourier transform of $u(x)$ (one-dimensional notation is used, extensions to two dimensions being straight-forward).

There are two difficulties associated with Eq. (1) and its application: (a) the definitions of W_o and B_o generally lack precision, especially since it is mathematically impossible for both $u(x)$ and $U(\nu)$ to have compact support; and (b) the effect of too few samples, as manifested by aliasing in the reconstruction of $u(x)$ from a set of discrete samples, is difficult to quantify. Propagation of $u(x)$ introduces additional complications since, through diffraction, it leads to a spreading of the wave—and, thus, to an increase in spatial extent $W(z)$ —that is often difficult to quantify. As noted in Ref. [1], the Wigner phase-space diagram can provide insight into the spreading of a wave as it propagates, but it does not provide a clear means for specifying the sample rate and number of samples appropriate for a given wave $u(x)$ and propagation distance z.

This paper has two objectives: (1) to provide means for selecting W_o and B_o in Eq. (1) that relates quantitatively to errors in the reconstruction of $u(x)$ from its sample values, and (2) to present a means for specifying the number of samples $N(z)$ required for numerical propagation of $u(x)$ through distance z, for $z_1 < z < z_2$, in the case where $|u(x, z_1)|$ and $|u(x, z_2)|$ are both known.

The paper proceeds as follows. First, the width W_o and spatial frequency bandwidth B_o of wave $u(x)$ are defined in terms of a fraction-of-signal-energy metric η. Next, it is shown that the fractional mean-square error in the continuous reconstruction $\tilde{u}(x)$ obtained from signal samples u_n is expressible in terms of η. Bounds on $\eta(z)$ are then established for propagation distance z satisfying $z_1 \leq z \leq z_2$, first for a special "light tube" case [2], then for a general case. The application of the energy-confining "light tube" to efficient numerical simulation of wave propagation is then discussed.

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[1] A. W. Lohmann, R. G. Dorsch, D. Mendlovic, Z. Zalevsky, and C. Ferreira, "Space-bandwidth product of optical signals and systems," *J. Opt. Soc. Am. A*, Vol. 13, pp. 470-473 (1996).
[2] William T. Rhodes, "Light Tubes, Wigner Diagrams, and Optical Wave Propagation Simulation," in *Optical Information Processing: A Tribute to Adolf Lohmann*, H. John Caulfield, Editor (SPIE Press, Bellingham, 2002), Chapter 15 (pp. 343-356).